

Streeter-Phelps equation

... (8.2) or following Streeter-Phelps equation ; i.e.,

$$D_t = \frac{K_D \cdot L}{K_R - K_D} \left[(10)^{-K_D \cdot t} - (10)^{-K_R \cdot t} \right] + \left[D_0 \times (10)^{-K_R \cdot t} \right] \quad \dots(8.3)$$

where D_t = the D.O. deficit in mg/l after t days.

L = Ultimate first stage B.O.D. of the mix at the point of waste discharge in mg/l.

D_0 = Initial oxygen deficit of the mix at the mixing point in mg/l.

K_D = De-oxygenation coefficient for the wastewater, which can be considered as equal to the BOD rate constant* deter-

K_R^{**} = Re-oxygenation coefficient for the stream.

The critical time (t_c):

$$t_c = \frac{1}{K_D(f-1)} \log \left[\left\{ 1 - (f-1) \frac{D_0}{L} \right\} f \right]$$

the critical or max. oxygen deficit

Self Purification Constant $\left(r = \frac{K_D}{K_D} \right)$

$$D_c = \frac{L}{f} \left[10 \right]^{-K_D t_c}$$

$$\left(\frac{L}{D_c \cdot f} \right)^{f-1} = f \left[1 - (f-1) \frac{D_0}{L} \right]$$

Example 8.1. The sewage of a town is to be discharged into a river stream. The quantity of sewage produced per day is 8 million litres, and its B.O.D. is 250 mg/l. If the discharge in the river is 200 l/s and if its B.O.D. is 6 mg/l, find out the B.O.D. of the diluted water.

Solution. Sewage discharge = Q_S

$$= \frac{8 \times 10^6}{24 \times 60 \times 60} \text{ l/s.} = 92.59 \text{ l/s.}$$

Discharge of the river = $Q_R = 200 \text{ l/s}$

B.O.D. of sewage = $C_S = 250 \text{ mg/l}$

B.O.D. of river = $C_R = 6 \text{ mg/l.}$

Using equation (8.1), we have

B.O.D. of the diluted mixture

$$= C = \frac{C_S \cdot Q_S + C_R \cdot Q_R}{Q_S + Q_R}$$

or

$$C = \frac{250 \times 92.59 + 6 \times 200}{92.59 + 200}$$
$$= 83.21 \text{ mg/l. Ans.}$$

Example 8.2. *In the above example, what should be the river discharge, if it is desired to reduce the B.O.D. of diluted water to 20 mg/l.*

Solution. Here $C = 20 \text{ mg/l}$

$$\therefore 20 = \frac{250 \times 92.59 + 6 \times Q_R}{92.59 + Q_R}$$

or

$$Q_R = 1521 \text{ l/s. Ans.}$$

Example 8.3. A city discharges 1500 litres per second of sewage into a stream whose minimum rate of flow is 6000 litres per second. The temperature of sewage as well as water is 20°C. The 5 day B.O.D. at 20°C for sewage is 200 mg/l and that of river water is 1 mg/l. The D.O. content of sewage is zero, and that of the stream is 90% of the saturation D.O. If the minimum D.O. to be maintained in the stream is 4.5 mg/l, find out the degree of sewage treatment required. Assume the de-oxygenation coefficient as 0.1, and re-oxygenation coefficient as 0.3.

Solution. From the table given at the end of the book in Appendix A-3, the value of saturation D.O. at 20°C is found out as 9.17 mg/l.

D.O. content of the stream

$$= 90\% \text{ of the saturation D.O.}$$

$$= \frac{90}{100} \times 9.17 = 8.25 \text{ mg/l.}$$

D.O. of mix at the start point (i.e. at $t = 0$)

$$= \frac{8.25 \times 6000 + 0 \times 1500}{6000 + 1500}$$

(\because D.O. of sewage is zero)

$$= 6.6 \text{ mg/l}$$

$$\begin{aligned}
 \therefore D_0 &= \text{initial D.O. deficit} \\
 &= [\text{Saturation D.O. at mix. temp.} - \text{D.O. of mix.}] \\
 &= 9.17 - 6.6 = 2.57 \text{ mg/l} \quad (\text{Assume instantaneous mixing})
 \end{aligned}$$

$$\begin{aligned}
 \text{Minimum D.O. to be maintained in the stream} \\
 &= 4.5 \text{ mg/l.}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Max. permissible saturation deficit (i.e., critical D.O. deficit)} \\
 &= D_c = 9.17 - 4.5 \\
 &= 4.67 \text{ mg/l.}
 \end{aligned}$$

Now, using equations (8.11), the first stage B.O.D. of mixture of sewage and stream (L) is given by

$$\left[\frac{L}{D_c f} \right]^{f-1} = f \left[1 - (f-1) \frac{D_0}{L} \right]$$

Substituting the values as :

$$D_0 = 2.57 \text{ mg/l} \quad \text{and} \quad D_c = 4.67 \text{ mg/l}$$

$$f = \frac{K_R}{K_D} = \frac{0.3}{0.1} = 3$$

we get $\left[\frac{L}{4.67 \times 3} \right]^{3-1} = 3 \left[1 - (3-1) \frac{2.57}{L} \right]$

or $\left[\frac{L}{14.01} \right]^2 = 3 \left[1 - \frac{5.14}{L} \right]$

Solving by hit and trial, we get the value

$$L = 21.1 \text{ mg/l}$$

Now, using $Y_t = L \left[1 - 10^{-K_D \cdot t} \right]$, we have

Max. permissible 5 day B.O.D. of the mix (at 20°C)

$$Y_5 = 21.1 \left[1 - 10^{-0.1 \times 5} \right] \quad (\text{where } K_D \text{ at } 20^\circ\text{C} = 0.1)$$
$$= 14.43 \text{ mg/l.}$$

Now, using equation (8.1), we have

$$C = \frac{C_S Q_S + C_R Q_R}{Q_S + Q_R}$$

where C stands for concentrations of B.O.D.

Substituting the values, we get

$$14.43 = \frac{C_S \times 1500 + 1 \times 6000}{1500 + 6000}$$

where C_S will represent the permissible B.O.D.₅ (at 20°C of course) of the discharged wastewater.

Solving, we get

$$C_S = 68.16 \text{ mg/l.}$$

∴ Degree of treatment required (per cent)

$$= \frac{\text{Original B.O.D. of sewage} - \text{Permissible B.O.D.}}{\text{Original B.O.D.}} \times 100$$

$$= \frac{200 - 68.16}{200} = \frac{131.84}{200}$$

$$= 65.9\%. \text{ Ans.}$$

Example 8.4. A city discharges 100 cumecs of sewage into a river, which is fully saturated with oxygen and flowing at the rate of 1500 cumecs during its lean days with a velocity of 0.1 m/sec. The 5-days BOD of sewage at the given temperature is 280 mg/l. Find when and where the critical D.O. deficit will occur in the downstream portion of the river, and what is its amount. Assume coefficient of purification of the stream (f) as 4.0, and coefficient of de-oxygenation (K_D) as 0.1.

Solution. The initial D.O. of river

$$= \text{Saturation D.O. at the given temp.} = 9.2 \text{ mg/l (say)}$$

D.O. of mix at $t = 0$ i.e., at start

$$= \frac{9.2 \times 1500 + 0 \times 100}{1500 \times 100} \quad (\text{assuming that D.O. of sewage is nil})$$

$$= 8.62 \text{ mg/l}$$

Initial D.O. deficit of the stream

$$= D_0 = 9.2 - 8.62 = 0.58 \text{ mg/l}$$

Also, 5-day BOD of the mixture of sewage and stream is given by

$$C = \frac{C_S Q_S + C_R Q_R}{Q_S + Q_R}$$

$$= \frac{280 \times 100 + 0 \times 1500}{100 + 1500}$$

$$= \frac{280 \times 100}{1600} = 17.5 \text{ mg/l.}$$

$$\text{BOD of mix at the given temp.} = Y_5 = 17.5 \text{ mg/l}$$

$$= \frac{280 \times 100}{1600} = 17.5 \text{ mg/l.}$$

\therefore 5 day BOD of mix at the given temp. = $Y_5 = 17.5 \text{ mg/l}$

$$Y_5 = L \left[1 - (10)^{-K_D \times 5} \right] \text{ and } K_D = 0.1 \text{ (at } 20^\circ\text{C)}$$

\therefore The ultimate BOD of the mix (i.e. L)

$$= \frac{17.5}{0.684} = 25.58 \text{ mg/l.}$$

Now, using equation (8.11), we have

$$\left[\frac{L}{D_c \cdot f} \right]^{f-1} = f \left[1 - (f-1) \frac{D_0}{L} \right]$$

$$\left[\frac{25.58}{D_c \times 4} \right]^3 = 4 \left[1 - \frac{3 \times 0.58}{25.58} \right]$$

or $D_c = 4.12 \text{ mg/l. Ans.}$

Now, from equation (8.8), we have

$$t_c = \frac{1}{K_D(f-1)} \log_{10} \left[f \left\{ 1 - (f-1) \frac{D_0}{L} \right\} \right]$$

or
$$t_c = \frac{1}{0.1(4-1)} \log_{10} \left[4 \times \left\{ 1 - \frac{3 \times 0.58}{25.58} \right\} \right]$$

$$= \frac{1}{0.3} \times 0.571 = 1.905 \text{ days.}$$

Now,
$$\begin{aligned} \text{distance} &= \text{Velocity of river} \times \text{Travel time} \\ &= 0.1 \text{ m/sec} \times (1.905 \times 24 \times 60 \times 60 \text{ sec}) \\ &= 16,460 \text{ m} = 16.46 \text{ km} \end{aligned}$$

Hence, the most critical deficit will occur after 1.905 days and at point 16.46 km downstream of the point of sewage disposal. **Ans.**

Example 8.5. A town with a population of 30,000 has to design a sewage treatment plant to handle industrial as well as domestic wastewaters of the town. A sanitary survey revealed the following :

Dairy wastes of 3 million litres per day with BOD of 1100 mg/l, and sugar mill waste of 2.4 million litres per day with BOD of 1500 mg/l are produced. In addition, domestic sewage is produced at the rate of 240 litres per capita per day. The per capita BOD of domestic sewage being 72 gm/day. An overall expansion factor of 10 per cent to be provided. The sewage effluents are to be discharged to a river stream with a minimum dry weather flow of 4500 litre per second and a saturation dissolved oxygen content of 9 mg/l. It is necessary to maintain a dissolved oxygen content of 4 mg/l in the stream. Determine the degree of treatment required to be given to the sewage. Assume suitable values of coefficients of de-oxygenation and re-oxygenation.

Solution. Per capita B.O.D. of the domestic sewage
= 72 gm/day = 72×1000 mg/day.

The per capita sewage produced
= 240 litre/day.

∴ B.O.D. per litre of the domestic sewage

$$= \frac{72 \times 1000}{240} \text{ mg/l.} = 300 \text{ mg/l.}$$

Amount of domestic wastewater produced per day

$$= 30,000 \times 240 \text{ litres} = 7.2 \text{ million litres}$$

Net B.O.D. of all wastewaters (i.e. domestic + industrial)

$$= \left[\frac{7.2 \times 300 + 3 \times 1100 + 2.4 \times 1500}{7.2 + 3 + 2.4} \right] = 719 \text{ mg/l.}$$

Total wastewater discharge

$$= \frac{\text{Vol. of wastewaters entering per day}}{\text{No. of secs in 1 day}}$$

$$= \frac{3 \text{ Ml} + 2.4 \text{ Ml} + 7.2 \text{ Ml}}{1 \times 24 \times 60 \times 60 \text{ sec}}$$

$$= \frac{12.6 \times 10^6}{24 \times 3600} \text{ l/s} = 145.8 \text{ l/s}$$

$$= \frac{24 \times 3600}{24 \times 3600}$$

Total wastewater discharge with 10% expansion factor

$$= 1.1 \times 145.8 \text{ l/s} = 160 \text{ l/s}$$

Initial D.O. of saturated stream water

$$= 9 \text{ mg/l (i.e. saturation D.O. as given)}$$

∴ D.O. of mixture at $t = 0$ i.e., at start point

$$= \frac{\text{D.O. of river} \times Q_R + \text{D.O. of sewage} \times Q_S}{Q_R + Q_S}$$

$$= \frac{9 \times 4500 + 0 \times 160}{4500 + 160}$$

(Assuming that the D.O. of wastewaters is nil)

$$= 8.69 \text{ mg/l}$$

∴ Initial D.O. deficit

$$= D_0 = 9 - 8.69$$

$$= 0.31 \text{ mg/l}$$

(assuming instantaneous mixing)

Also, critical D.O. deficit, i.e. allowable max. D.O. deficit

$$= D_c = 9 - 4.0$$

$$= 5 \text{ mg/l}$$

Now, using eqn. (8.11), we have

$$\left[\frac{D_c - D}{D_c} \right]^{f-1} = f \left[1 - (f - 1) \frac{D_0}{L} \right]$$

where $D_c = 5 \text{ mg/l}$,

$D_0 = 0.31 \text{ mg/l}$,

$K_D = 0.1$; $K_R = 0.3$; $f = 3$

(assumed values at mix. temp.)

$$\therefore \left[\frac{L}{5 \times 3} \right]^2 = 3 \left[1 - \frac{2 \times 0.31}{L} \right]$$

Solving by hit and trial,

$$L = 25.65 \text{ mg/l}$$

Max. permissible 5 day B.O.D. of mix at mix temp.

$$= Y_5 = L \left[1 - (10)^{-0.1 \times 5} \right] \quad (K_D \text{ at mix temp. is assumed} = 0.1)$$

$$= 0.684 L$$

$$= 0.684 \times 25.65 = 17.54 \text{ mg/l}$$

Using eqn. (8.1) as

$$C = \frac{C_S Q_S + C_R Q_R}{Q_S + Q_R}$$

$$\text{we get } 17.54 = \frac{C_S + 160 + 0 \times 4500}{160 + 4500}$$

where C_S = Max. permissible B.O.D.₅ of waste-waters
or $C_S = 511 \text{ mg/l.}$

$$\therefore \text{Permissible B.O.D. of wastewaters} \\ = 511 \text{ mg/l}$$

$$\text{Initial B.O.D. of city wastewaters} \\ = 719 \text{ mg/l}$$

$$\therefore \text{Degree of treatment reqd.} \\ = \frac{719 - 511}{719} = 28.93\% \quad \text{Ans.}$$

Example 8.6. In the previous example, determine what should be the dilution ratio if no treatment was required, and thus determine the river discharge for such a condition.

Solution. When no treatment is required, the value of max. permissible BOD_5 of wastewaters, i.e. C_S should be 719. Q_R can then be determined as :

$$17.54 = \frac{719 \times 160 + D \times Q_R}{160 + Q_R}$$

or $17.54 (160 + Q_R) = 719 \times 160$

or $160 + Q_R = \frac{719 \times 160}{17.54} = 6559$

or $Q_R = 6399 \text{ l/s (say). Ans.}$

$$\text{Dilution ratio} = \frac{6399}{160} = 39.99 ; \text{ Say 40 times.}$$

Hence when the dilution ratio is 40 and the minimum river discharge is 6400 l/s, no treatment will be required. **Ans.**

[Note. Strictly speaking, when Q_R increases, D_0 will reduce, increasing Y_5 , needing repeat of calculations, to obtain precise results. But the effect will be very small and on safer side, and hence is generally ignored.

Example 8.7. A waste water effluent of 560 l/s with a BOD = 50 mg/l, DO = 3.0 mg/l and temperature of 23°C enters a river where the flow is 28 m³/sec, and BOD = 4.0 mg/l, DO = 8.2 mg/l, and temperature of 17°C. k_1 of the waste is 0.10 per day at 20°C. The vel. of water in the river downstream is 0.18 m/s and depth of 1.2 m. Determine the following after mixing of waste water with the river water :

- (i) Combined discharge ;
- (ii) BOD ;
- (iii) DO ; and
- (iv) Temperature.

(Civil Services, 1981)

Solution.

<i>Particulars of Sewage thrown</i>	<i>Particulars of River</i>
$Q_S = 560 \text{ l/s}$ $= 0.56 \text{ m}^3/\text{sec}$	$Q_R = 28 \text{ m}^3/\text{sec}$
Concentrations (C_S)	Concentrations (C_R)
BOD = 50 mg/l	BOD = 4.0 mg/l
DO = 3.0 mg/l	DO = 8.2 mg/l
Temp. = 23°C	Temp. = 17°C.
k_1 at 20° = 0.1 per day	

(i) *Combined discharge*

$$= Q_S + Q_R$$

$$= 0.56 + 28 = 28.56 \text{ m}^3/\text{sec. Ans.}$$

Now, using equation (8.1), for conc. of mix as

$$C = \frac{C_S \cdot Q_S + C_R \cdot Q_R}{Q_S + Q_R}, \text{ we have}$$

(ii) *BOD of mix*

$$= \frac{50 \times 0.56 + 4.0 \times 28}{0.56 + 28} = \frac{140}{28.56} = 4.9 \text{ mg/l.}$$

(iii) *DO of mix*

$$= \frac{3.0 \times 0.56 + 8.2 \times 28}{0.56 + 28} = 8.098 \text{ mg/l Ans.}$$

(iv) *Temp. of mix*

$$= \frac{23 \times 0.56 + 17 \times 28}{0.56 + 28} = 17.12^\circ\text{C. Ans.}$$

Example 8.8. 125 cumecs of sewage of a city is discharged in a perennial river which is fully saturated with oxygen and flows at a minimum rate of 1600 cumecs with a minimum velocity of 0.12 m/sec. If the 5 day BOD of the sewage is 300 mg/l, find out where the critical DO will occur in the river. Assume :

- (i) the coefficient of purification of the river as 4.0,
- (ii) the ultimate BOD as 125% of the 5 day BOD of the mixture of sewage and river water.
- (iii) Saturation DO of river = 9.2 mg/l

Solution. Saturation D.O. Concentration of the given river
= 9.2. (given in assumption iii)

The D.O. of the river at the mixing point after disposal of sewage (D)

$$= \frac{125 \times 0 + 1600 \times 9.2}{125 + 1600} = 8.53 \text{ mg/l}$$

Initial D.O. deficit (D_0) = $D_s - D$

$$= 9.2 - 8.53 = 0.67 \text{ mg/l}$$

BOD₅ of the river at the mixing point after disposal of sewage (Y_5)

$$= \frac{125 \times 300 + 1600 \times 0}{125 + 1600} = 21.74 \text{ mg/l}$$

The distance along the river, where the critical D.O. deficit will occur

$$= S = \text{Velocity} \times \text{Time}$$

$$= 0.12 \text{ m/sec} \times (1.354 \times 24 \times 3600 \text{ sec})$$

$$= 14.04 \text{ km ; Say 14 km}$$

Hence, critical D.O. deficit will occur at 14 km downstream of the sewage disposal point. **Ans.**

Example 8.9. A wastewater treatment plant disposes of its effluents into a stream at a point A. Characteristics of the stream at a location fairly upstream of A and of the effluent are as below:

Item	Units	Effluent	Stream
Flow	m^3/s	0.20	0.50
Dissolved oxygen	mg/l	2.00	8.00
Temperature	$^{\circ}\text{C}$	26	22
BOD_5 at 20°C	mg/l	40	3

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Flow	m^3/s	0.20	0.50
Dissolved oxygen	mg/l	2.00	8.00
Temperature	$^{\circ}C$	26	22
BOD_5 at $20^{\circ}C$	mg/l	40	3

Assume that the deoxygenation constant K_1 at $20^{\circ}C$ (base e) = $0.20 d^{-1}$ and the re-aeration constant K_2 at $20^{\circ}C$ (base e) = $0.40 d^{-1}$ for the mixture. Equilibrium concentration of dissolved oxygen C_s for the fresh water is as follows:

Temperature $^{\circ}C$	18	20	22	23	24	25	26
C_s (mg/l)	9.54	9.17	8.99	8.83	8.53	8.38	8.22

The velocity of the stream downstream of the point A is $0.2 m/s$. Determine the critical oxygen deficit and its location.

[Use temperature coefficients of 1.04 for K_1 and 1.02 for K_2]

Solution. K_1 at 20°C (base e)
 $= 0.2 \text{ d}^{-1} = 0.2 \text{ per day}$

$\therefore K_D$ at 20°C (base 10)
 $= \frac{K_1}{2.3} = 0.434 K_1$
 $= 0.434 \times 0.2 \text{ per day} = 0.087 \text{ per day.}$

Similarly, K_R at 20°C
 $= 0.434 \times 0.4 \text{ d}^{-1} = 0.174 \text{ per day.}$

The formulas to be used in this question for converting K_D and K_R at any other temperature ($T^\circ\text{C}$) will be

$$K_{D(T^\circ)} = K_{D(20^\circ)} \left[1.04 \right]^{T^\circ - 20^\circ}; \text{ and}$$

$$K_{R(T^\circ)} = K_{R(20^\circ)} \left[1.02 \right]^{T^\circ - 20^\circ} \quad (\text{as per the given values})$$

(i) We will now determine DO, BOD and temperature of mixture as below:

$$\text{DO of mixture} = \frac{\text{DO of sewage} \times Q_S + \text{DO of river} \times Q_R}{Q_S + Q_R}$$

$$= \frac{2 \times 0.20 + 8 \times 0.50}{0.20 + 0.50}$$

$$= 6.29 \text{ mg/l.}$$

BOD₅ of mixture

(i.e. 5 day BOD at 20°C)

$$= \frac{40 \times 0.20 + 3 \times 0.50}{0.20 + 0.50}$$

$$= 13.57 \text{ mg/l.}$$

Temperature of mixture

$$= \frac{26 \times 0.20 + 22 \times 0.50}{0.20 + 0.50} = 23.14^\circ\text{C}$$

(ii) Ultimate BOD of mixture (L)

$$L = \frac{Y_5 \text{ (i.e. 5 day BOD of mixture at } 20^\circ\text{C)}}{1 - (10)^{-K_D \times 5}}$$

where K_D is at $20^\circ\text{C} = 0.087$ per day

$$= \frac{13.57}{1 - (10)^{-0.087 \times 5}} = \frac{13.57}{0.633} = 21.46 \text{ mg/l.}$$

(iii) Initial D.O. Deficit of mixture,

$$\text{D.O. of mixture} = 6.29 \text{ mg/l}$$

$$\begin{aligned} \text{Saturation D.O. at mixture temperature of } 23.14^\circ\text{C} \\ = 8.79 \text{ (interpolated from given values)} \end{aligned}$$

$$\begin{aligned} \therefore D_0 &= \text{D.O. deficit} \\ &= 8.79 - 6.29 = 2.50 \text{ mg/L.} \end{aligned}$$

(iv) Corrected values of K_D and K_R are :

$$\begin{aligned} K_{D(23.14^\circ)} &= K_{D(20^\circ)} [1.04]^{T-20} \\ &= 0.087 [1.04]^{3.14} = 0.098 \end{aligned}$$

$$\begin{aligned} K_{R(23.14^\circ)} &= K_{R(20^\circ)} [1.02]^{T-20} \\ &= 0.174 [1.02]^{3.14} = 0.185 \end{aligned}$$

(v) The time (t_c) after which critical D.O. deficit (D_c) occurs is given by eqn. (8.8) as

$$t_c = \frac{1}{K_D(f-1)} \log_{10} \left[\left\{ 1 - (f-1) \frac{D_0}{L} \right\} f \right]$$

where $K_R = 0.185$
 $K_D = 0.098$

$$\therefore f = \frac{K_R}{K_D} = \frac{0.185}{0.098} = 1.888$$

$$L = 21.45 \text{ mg/l}$$

$$D_0 = 2.5 \text{ mg/l.}$$

$$\therefore t_c = \frac{1}{0.098(1.888 - 1)} \log_{10} \left[\left(1 - \frac{0.888 \times 2.5}{21.45} \right) 1.888 \right]$$
$$= \frac{1}{0.098(0.888)} \times 0.228 = 2.625 \text{ days.}$$

(vi) Now, $\text{Distance} = \text{Velocity} \times \text{Travel time}$
 $= 0.2 \text{ m/s} \times (2.625 \times 24 \times 60 \times 60 \text{ sec})$
 $= 45.36 \text{ km. Ans.}$

(vii) D_c is now given by eqn. (8.11) as

$$\left(\frac{L}{D_c \cdot f} \right)^{f-1} = f \left(1 - (f-1) \frac{D_0}{L} \right)$$

or $\left(\frac{21.45}{D_c \times 1.888} \right)^{0.888} = 1.888 \left(1 - \frac{0.888 \times 2.5}{21.45} \right)$

or $\frac{21.45}{1.888 D_c} = \left(1.692 \right)^{\frac{1}{0.888}} = (1.692)^{1.126} = 1.808$

or $D_c = \frac{21.45}{1.888 \times 1.808} = 6.28 \text{ mg/l.}$

Hence, the critical D.O. deficit equal to 6.28 mg/l occurs at 45.36 km downstream of A, after 2.625 days. **Ans.**

Example 8.10. A treated waste water is discharged at the rate of $1.5 \text{ m}^3/\text{sec}$ into a river of minimum flow $5 \text{ m}^3/\text{sec}$. The temperature of river flow and waste water flow may be assumed as 25°C . The BOD removal rate constant K_1 is $0.12/\text{d}$ (base 10). The BOD_5 at 25°C of the waste water is 200 mg/l , and that of the river water upstream of the wastewater outfall is 1 mg/l . The efficiency of waste water treatment is 80% . Evaluate the following :

- (i) BOD_5 at 25°C , if river water receives untreated waste water
- (ii) BOD_5 at 25°C if river water receives treated waste water
- (iii) ultimate BOD of the river water after it receives treated waste water.

(Civil Services, 1993)

Solution.

Discharge of waste water = $Q_w = 1.5 \text{ m}^3/\text{s}$

Discharge of river = $Q_R = 5 \text{ m}^3/\text{s}$

Temperature = $T = 25^\circ\text{C}$

$$K_{D(25^\circ)} = K_1 = 0.12/\text{d}.$$

C_w = Conc. of BOD_5 for untreated waste water = 200 mg/l

C_R = Conc. of BOD_5 for river water = 1 mg/l

Using eqn. (8.1),

(i) Conc. of BOD_5 of the mixture if untreated waste water is discharged into the river

$$= C = \frac{C_W \cdot Q_W + C_R \cdot Q_R}{Q_W + Q_R} = \frac{200 \times 1.5 + 1 \times 5}{1.5 + 5}$$

$$= 46.92 \text{ mg/l. Ans.}$$

(ii) BOD_5 of the treated wastewater is given by

$$C_{TW} = 20\% \text{ of the } BOD_5 \text{ of untreated wastewater}$$

(\because efficiency of wastewater treatment is 80%)

$$= 20\% \times C_W = 20\% \times 200 \text{ mg/l}$$

$$= 40 \text{ mg/l.}$$

BOD_5 of mixture if treated wastewater is discharged into the river

$$= C' = \frac{C_{TW} \cdot Q_W + C_R \cdot Q_R}{Q_W + Q_R}$$

$$= \frac{40 \times 1.5 + 1 \times 5}{1.5 + 5} = 10 \text{ mg/l. Ans.}$$

(iii) BOD_5 of river water after it receives treated wastewater
= 10 mg/l (as computed above)

Ultimate BOD of this mixture

$$= Y_u = L = ?$$

Using eqn. (7.16), we have

$$Y_{t,d_5} = L \left[1 - (10)^{-K_D \cdot t} \right]$$

$$\text{or } Y_5 = L \left[1 - (10)^{-0.12 \times 5} \right]$$

$$\text{or } 10 = L \left[1 - (10)^{-0.6} \right]$$

$$\text{or } L = 13.35 \text{ mg/l. Ans.}$$